RELAXATION OF INTERNAL TEMPERATURE AND VOLUME VISCOSITY

Domenico BRUNO and Vincent GIOVANGIGLI

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LIST OF TYPOGRAPHIC ERRORS

Equation (19) should be

$$\mathcal{S}^{\rm int} = nk_{\rm B}\Big(\frac{\overline{E}}{k_{\rm B}T^{\rm int}} - \log\frac{1}{Z^{\rm int}}\Big). \tag{19}$$

Equation (20) should be

$$G^{\rm tr} = k_{\rm B}T^{\rm tr}\log\frac{n}{Z^{\rm tr}}, \qquad G^{\rm int} = k_{\rm B}T^{\rm int}\log\frac{1}{Z^{\rm int}}, \tag{20}$$

Equation (22) should be

$$\mathrm{d}\mathcal{S} = \frac{nc^{\rm tr}}{T^{\rm tr}}\mathrm{d}T^{\rm tr} + \frac{nc^{\rm int}}{T^{\rm int}}\mathrm{d}T^{\rm int} + \Big(\frac{3}{2}k_{\rm B} + \frac{\overline{E}}{T^{\rm int}} - k_{\rm B}\log\frac{n}{Z^{\rm tr}Z^{\rm int}}\Big)\mathrm{d}n, \tag{22}$$

Equation (23) should be

$$\partial_t\mathcal{S}+\boldsymbol{\nabla}\cdot(\boldsymbol{v}\mathcal{S})+\boldsymbol{\nabla}\cdot\Big(\frac{\boldsymbol{Q}^{\rm tr}}{T^{\rm tr}}+\frac{\boldsymbol{Q}^{\rm int}}{T^{\rm int}}\Big) = -\frac{\boldsymbol{Q}^{\rm tr}\cdot\boldsymbol{\nabla}T^{\rm tr}}{{T^{\rm tr}}^2}-\frac{\boldsymbol{Q}^{\rm int}\cdot\boldsymbol{\nabla}T^{\rm int}}{{T^{\rm int}}^2}-\frac{\boldsymbol{\Pi}:\boldsymbol{\nabla}\boldsymbol{v}}{T^{\rm tr}}+\frac{\omega_1^{\rm int}(T^{\rm tr}-T^{\rm int})}{T^{\rm tr}T^{\rm int}}. \tag{23}$$

Equation (41) should be

$$\widetilde{\mathcal{J}}^{\rm sl,(0)} = -\frac{1}{2n^2[\![(\Delta E)^2\zeta]\!]}\sum_{\rm J,I',J'}\int f^{(0)}\widetilde{f}^{(0)}(\Delta E)\zeta g\sigma^{\rm IJI'J'}\mathrm{d}\widetilde{\boldsymbol{c}}\,\mathrm{d}\boldsymbol{e}', \tag{41}$$

Equation (76) should be

$$\psi^{\eta} = \frac{m}{k_{\rm B}\overline{T}}\big((\boldsymbol{c}-\boldsymbol{v})\otimes(\boldsymbol{c}-\boldsymbol{v}) - \tfrac{1}{3}(\boldsymbol{c}-\boldsymbol{v})\cdot(\boldsymbol{c}-\boldsymbol{v})\boldsymbol{I}\big), \tag{76}$$

Equation (77) should be

$$\psi^{\lambda^{\rm tr+rap}} = \big(\tfrac{5}{2}k_{\rm B}\overline{T} - \tfrac{1}{2}m(\boldsymbol{c}-\boldsymbol{v})\cdot(\boldsymbol{c}-\boldsymbol{v}) + \overline{E}^{\rm rap} - E_{\rm I}^{\rm rap}\big)(\boldsymbol{c}-\boldsymbol{v}), \tag{77}$$

Equation (81) should be

$$\widetilde{\mathcal{J}}^{\rm sl,(0)} = -\frac{1}{2n^2[\![(\Delta E^{\rm sl})^2\zeta^{\rm sl}]\!]}\sum_{\rm J,I',J'}\int f^{(0)}\widetilde{f}^{(0)}(\Delta E^{\rm sl})\zeta^{\rm sl}g\sigma^{\rm IJI'J'}\mathrm{d}\widetilde{\boldsymbol{c}}\,\mathrm{d}\boldsymbol{e}', \tag{81}$$

Equation (104) should be

$$\widehat{p}^{\mathrm{rel}} = -\frac{(k_{\mathrm{B}}\overline{T})^3 c^{\mathrm{rap}}}{p(c^{\mathrm{tr}}+c^{\mathrm{rap}})^2}\,\frac{c^{\mathrm{rap}}[\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]+(c^{\mathrm{tr}}+c^{\mathrm{rap}})[\![(\Delta E^{\mathrm{rap}})(\Delta E^{\mathrm{sl}})\zeta^{\mathrm{sl}}]\!]}{2[\![(\Delta E^{\mathrm{rap}})^2]\!][\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]}. \tag{104}$$

Equation (106) should be

$$\widehat{p}^{\mathrm{rel}} = \frac{(k_{\mathrm{B}}\overline{T})^3 c^{\mathrm{sl}} c^{\mathrm{rap}}}{(c^{\mathrm{tr}}+c^{\mathrm{rap}})^2 c_{\mathrm{v}}}\,\frac{c^{\mathrm{rap}}[\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]+(c^{\mathrm{tr}}+c^{\mathrm{rap}})[\![(\Delta E^{\mathrm{rap}})(\Delta E^{\mathrm{sl}})\zeta^{\mathrm{sl}}]\!]}{2[\![(\Delta E^{\mathrm{rap}})^2]\!][\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]}, \tag{106}$$

Equation (107) should be

$$\langle\!\langle f^{(0)}\phi^{0010}, \mathcal{W}^{\mathrm{sl}}\rangle\!\rangle = -\frac{2n^2}{k_{\mathrm{B}}\overline{T}}\Big([\![(\Delta E^{\mathrm{sl}})(\Delta E^{\mathrm{sl}}+\Delta E^{\mathrm{rap}})]\!]-2[\![(\Delta E^{\mathrm{sl}})^2\phi^{0010}\zeta^{\mathrm{sl}}]\!]\frac{\overline{T}-T^{\mathrm{sl}}}{T^{\mathrm{sl}}}\Big), \tag{107}$$

Equation (108) should be

$$\langle\!\langle f^{(0)}\phi^{0001\mathrm{rap}}, \mathcal{W}^{\mathrm{sl}}\rangle\!\rangle = \frac{2n^2}{k_{\mathrm{B}}\overline{T}}\Big([\![(\Delta E^{\mathrm{sl}})(\Delta E^{\mathrm{rap}})]\!]+2[\![(\Delta E^{\mathrm{sl}})^2\phi^{0001\mathrm{rap}}\zeta^{\mathrm{sl}}]\!]\frac{\overline{T}-T^{\mathrm{sl}}}{T^{\mathrm{sl}}}\Big). \tag{108}$$

Equation (110) should be

$$\langle\!\langle f^{(0)}\phi^{0010}, \mathcal{W}^{\mathrm{sl}}\rangle\!\rangle \approx -\frac{2n^2}{k_{\mathrm{B}}\overline{T}}[\![(\Delta E^{\mathrm{sl}})(\Delta E^{\mathrm{sl}}+\Delta E^{\mathrm{rap}})\zeta^{\mathrm{sl}}]\!], \tag{110}$$

Equation (111) should be

$$\langle\!\langle f^{(0)}\phi^{0001\mathrm{rap}}, \mathcal{W}^{\mathrm{sl}}\rangle\!\rangle \approx \frac{2n^2}{k_{\mathrm{B}}\overline{T}}[\![(\Delta E^{\mathrm{sl}})(\Delta E^{\mathrm{rap}})\zeta^{\mathrm{sl}}]\!].$$

Equation (117) should be

$$\phi^{\omega} = \frac{1}{p^2}\frac{1}{c^{\mathrm{tr}}+c^{\mathrm{rap}}}\frac{(k_{\mathrm{B}}\overline{T})^3}{2[\![(\Delta E^{\mathrm{rap}})^2]\!]}\Big(\frac{[\![(\Delta E^{\mathrm{sl}})(\Delta E^{\mathrm{rap}})\zeta^{\mathrm{sl}}]\!]}{[\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]}+\frac{c^{\mathrm{rap}}}{c^{\mathrm{tr}}+c^{\mathrm{rap}}}\Big)(c^{\mathrm{rap}}\,\phi^{0010}-c^{\mathrm{tr}}\,\phi^{0001\mathrm{rap}}). \tag{117}$$

The second line of Equation (121) should be

$$-\frac{c^{\mathrm{sl}}}{c_{\mathrm{v}}}\Big(\frac{c^{\mathrm{rap}}}{c^{\mathrm{tr}}+c^{\mathrm{rap}}}\Big)^2\frac{(k_{\mathrm{B}}\overline{T})^3}{2[\![(\Delta E^{\mathrm{rap}})^2]\!]}-\frac{c^{\mathrm{rap}}c^{\mathrm{sl}}}{(c^{\mathrm{tr}}+c^{\mathrm{rap}})c_{\mathrm{v}}}\frac{(k_{\mathrm{B}}\overline{T})^3[\![(\Delta E^{\mathrm{rap}})(\Delta E^{\mathrm{sl}})\zeta^{\mathrm{sl}}]\!]}{2[\![(\Delta E^{\mathrm{rap}})^2]\!][\![(\Delta E^{\mathrm{sl}})^2\zeta^{\mathrm{sl}}]\!]}$$

so that the resulting volume viscosity reads

$$
\begin{aligned}
\kappa = \quad & \Big(\frac{c^{\rm rap}}{c^{\rm tr}+c^{\rm rap}}\Big)^2 \frac{(k_{\rm B}\overline{T})^3}{2[\![(\Delta E^{\rm rap})^2]\!]} \\
& - \frac{c^{\rm sl}}{c_{\rm v}}\Big(\frac{c^{\rm rap}}{c^{\rm tr}+c^{\rm rap}}\Big)^2 \frac{(k_{\rm B}\overline{T})^3}{2[\![(\Delta E^{\rm rap})^2]\!]} - \frac{c^{\rm rap}c^{\rm sl}}{(c^{\rm tr}+c^{\rm rap})c_{\rm v}} \frac{(k_{\rm B}\overline{T})^3[\![(\Delta E^{\rm rap})(\Delta E^{\rm sl})\zeta^{\rm sl}]\!]}{2[\![(\Delta E^{\rm rap})^2]\!][\![(\Delta E^{\rm sl})^2\zeta^{\rm sl}]\!]} \\
& + \frac{c^{\rm sl}\widetilde{c}^{\rm sl}}{c_{\rm v}\widetilde{c}_{\rm v}} \frac{{k_{\rm B}}^3(\overline{T})^2 T^{\rm sl}}{2[\![\Delta E^{\rm sl})^2\zeta^{\rm sl}]\!]} \\
& - \frac{\widetilde{c}^{\rm sl}}{\widetilde{c}_{\rm v}}\Big(\frac{c^{\rm rap}}{c^{\rm tr}+c^{\rm rap}}\Big)^2 \frac{k_{\rm B}^3\overline{T}^2 T^{\rm sl}}{2[\![(\Delta E^{\rm rap})^2]\!]} - \frac{c^{\rm rap}\widetilde{c}^{\rm sl}}{(c^{\rm tr}+c^{\rm rap})\widetilde{c}_{\rm v}} \frac{k_{\rm B}^3\overline{T}^2 T^{\rm sl}[\![(\Delta E^{\rm sl})(\Delta E^{\rm rap})]\!]}{2[\![(\Delta E^{\rm rap})^2]\!][\![(\Delta E^{\rm sl})^2\zeta^{\rm sl}]\!]} \\
& + \frac{c^{\rm sl}\widetilde{c}^{\rm sl}}{c_{\rm v}\widetilde{c}_{\rm v}}\Big(\frac{c^{\rm rap}}{c^{\rm tr}+c^{\rm rap}}\Big)^2 \frac{k_{\rm B}^3\overline{T}^2 T^{\rm sl}}{2[\![(\Delta E^{\rm rap})^2]\!]} + 2\frac{c^{\rm rap}c^{\rm sl}\widetilde{c}^{\rm sl}}{(c^{\rm tr}+c^{\rm rap})c_{\rm v}\widetilde{c}_{\rm v}} \frac{k_{\rm B}^3\overline{T}^2 T^{\rm sl}[\![(\Delta E^{\rm sl})(\Delta E^{\rm rap})]\!]}{2[\![(\Delta E^{\rm rap})^2]\!][\![(\Delta E^{\rm sl})^2\zeta^{\rm sl}]\!]}. \qquad (121)
\end{aligned}
$$