Erratum

A few errors and typos have been identified in the book *Multicomponent Flow Modeling* and are listed here. The author is very grateful to colleagues for their help in spotting these misprints.

Chapter 2

- On page 32 Reference [CC70] is out of order. On page 34, Reference [Fei95] should be deleted (it is only mentioned in Chapter 6 and not in Chapter 2).

Chapter 3

- On page 47, Equation (3.3.23) should be

$$Y_k = Y_k(t, y), \qquad k \in S. \tag{3.3.23}$$

- On page 48, Equation (3.3.31) should be

$$\rho \partial_t \widehat{u} + \rho v \partial_y \widehat{u} = \partial_y \big(\eta \partial_y \widehat{u} \big) + (J - \rho \widehat{u}^2). \tag{3.3.31}$$

- On page 50, Equation (3.3.47) should be

$$\widetilde{u}(t, +\infty) = 1, \tag{3.3.47}$$

- On page 58, in Reference [Spo97], at line 7, rediction should be reduction.

Chapter 4

- On page 76, the mixture specific heat at constant volume per unit molecule should be $\mathfrak{c}_v$ and (4.6.19) should be

$$
\Psi_i^r = \frac{\widetilde{\Psi}_i^r}{f_i^0} - \frac{\nu_{ir}^{\mathrm{b}} - \nu_{ir}^{\mathrm{f}}}{n_i} - \frac{\sum_{j \in S}(\frac{3}{2}k_{\mathrm{B}}T + \overline{E}_j)(\nu_{jr}^{\mathrm{b}} - \nu_{jr}^{\mathrm{f}})}{p\mathfrak{c}_v T}
$$
$$
\times \left(\tfrac{3}{2}k_{\mathrm{B}}T - \tfrac{1}{2}\mathfrak{m}_i(\boldsymbol{c}_i - \boldsymbol{v})\!\cdot\!(\boldsymbol{c}_i - \boldsymbol{v}) + \overline{E}_i - E_{iI}\right), \quad (4.6.19)
$$

- On page 76, Equation (4.6.23) should be

$$
\phi_i^{\mathfrak{C}} = \sum_{r \in R} \phi_i^r \, \overline{\tau}_r. \tag{4.6.23}
$$

- On page 77, second line, $\langle\!\langle \Psi^\mu, \psi^l \rangle\!\rangle = 0$ should be $\langle\!\langle f^0 \Psi^\mu, \psi^l \rangle\!\rangle = 0$.

- On page 86, Equation (4.7.3) should be

$$
[\![\xi^{\mathfrak{p}k}, \phi^\mu]\!] = \langle\!\langle \xi^{\mathfrak{p}k}, f^0 \Psi^\mu \rangle\!\rangle, \qquad (\mathfrak{p}, k) \in \mathcal{B}^\mu, \tag{4.7.3}
$$

- On page 87, Equation (4.7.7) should be

$$
[\![\phi^\mu, \phi^\mu]\!] = \langle\!\langle \Psi^\mu, f^0 \phi^\mu \rangle\!\rangle = \sum_{(\mathfrak{p},k) \in \mathcal{B}^\mu} \alpha_k^{\mathfrak{p}\mu} \beta_k^{\mathfrak{p}\mu}, \tag{4.7.7}
$$

- On page 88, five lines below equation (4.7.12), $\phi^\mu = (phi_i^\mu)_{i \in S}$ should be $\phi^\mu = (\phi_i^\mu)_{i \in S}$.

Chapter 5

- On page 102, Equation (5.3.4) should be

$$
\xi = 1 + \frac{1}{4}\alpha_l^*(\mu_k^*)^2 \sqrt{\frac{\epsilon_k}{\epsilon_l}}, \tag{5.3.4}
$$

- On page 106, the $\mathcal{H}$ matrix should be

$$
\mathcal{H}_{kk} = \sum_{\substack{l \in S \\ l \neq k}} \frac{2k_{\mathrm{B}}T}{p\mathcal{D}_{kl}^{\mathrm{bin}}} \frac{X_k X_l}{\mathfrak{m}_k + \mathfrak{m}_l} \left[1 + \frac{3}{5}\frac{\mathfrak{m}_l}{\mathfrak{m}_k}\overline{A}_{kl}\right] + \frac{X_k^2}{\eta_k}, \qquad k \in S, \quad (5.4.2)
$$

$$
\mathcal{H}_{kl} = \frac{2k_{\mathrm{B}}T}{p\mathcal{D}_{kl}^{\mathrm{bin}}} \frac{X_k X_l}{\mathfrak{m}_k + \mathfrak{m}_l} \left[-1 + \frac{3}{5}\overline{A}_{kl}\right], \qquad k, l \in S, \quad k \neq l. \quad (5.4.3)
$$

This error—the missing factors 2—also appears in Reference [EG98b] mentioned in Chapter 12.

- On page 107, the volume viscosity κ_k of the kth species should be

$$\kappa_k = \frac{k_{\text{B}}\pi}{4}\Big(\frac{\mathfrak{c}_k^{\text{int}}}{\mathfrak{c}_{vk}}\Big)^2\frac{\xi_k^{\text{int}}}{\mathfrak{c}_k^{\text{int}}}\eta_k, \qquad k \in S_{\text{pol}}. \tag{5.5.2}$$

- On page 109, before equation (5.6.7), the matrix $\mathcal{M}$ should be $\mathcal{M} = \text{diag}(X_1/D_1^*, \ldots, X_n/D_n^*)$.

- On page 109, after equation (5.6.7), at line -8, the projector P should be $P = I - \mathcal{U}{\otimes}Y/\langle\mathcal{U}, Y\rangle$.

Chapter 6

- On page 132, in the line after Equation (6.3.30), $(e, 1/\rho, Y)$ should be $(e, 1/\rho, Y)^t$.

Chapter 7

- On page 161, in the first paragraph, the first sentence should be "For point centers of repulsion, we have $\varphi = T^{(\nu+4)/2\nu}$, where ν varies from $\nu = 4$—Maxwell molecules—to $+\infty$ and the temperature dependence varies, respectively, between $\varphi = T$ and $\varphi = T^{1/2}$ [FK72]."

- On page 166, there is an 'unidentified flying comma' in Equation (7.3.5).

- On page 188, the line after equation (7.7.18), $\text{Srd}(T)$ and $\text{Srd}(PT)$ should be $\text{Srd}(\mathcal{T})$ and $\text{Srd}(P\mathcal{T})$.

Chapter 8

- On page 218, in equation (8.7.35), the lower right coefficient of the matrix $\overline{\mathcal{A}}_0$ should be

$$\frac{\Upsilon_e' - \rho T\boldsymbol{v}{\cdot}\boldsymbol{v}}{T^4}.$$

Chapter 9

- On page 223, at line -2, $\mathbb{R}^d$ should be $\mathbb{R}^{n^*}$.

- On page 225, first line, the eigenvalues $\lambda^*(\zeta, w)$ have a negative real part for $\zeta \in i\,\mathbb{R}\backslash\{0\}$ and $w \in \Sigma^{d-1}$ [SK85].

- On page 225, third line of the second paragraph, "$j \in C$" sould be "$j \in C^*$" and, similarly, at the last line of the same paragraph, the matrix $K(w)$ should be $K(w) = \sum_{j \in C^*} K^j w_j$. The same misprint appears on pages 230 and 231 in Equation (9.4.16) and following.

- On page 227, properties (9.4.5) should be

$$\begin{cases} \partial_x W_I^* \in L^2(0, \infty; W_2^{l-1}(\mathbb{R}^d)), \\ \partial_x W_{II}^* \in L^2(0, \infty; W_2^l(\mathbb{R}^d)). \end{cases} \tag{9.4.5}$$

Similarly, on page 242, (9.7.9) should be

$$\begin{cases} \partial_x W_I \in L^2(0, \infty; W_2^{l-1}(\mathbb{R}^d)), \\ \partial_x W_{II} \in L^2(0, \infty; W_2^l(\mathbb{R}^d)). \end{cases} \tag{9.7.9}$$

- On page 232, the last line should be

$$\sup_{\mathbb{R}^d} |W^*(t) - W^{*e}| \;\leq\; C \, \|\partial_x^{l-1} W^*(t)\|_{0,2}^a \, \|W^*(t) - W^{*e}\|_{0,2}^{1-a},$$

- On page 240, line -10, x^t should be x in the scalar product of the left hand side.

Chapter 10

- On page 248, Equation (10.2.16) should be

$$\xi^{\mathrm{a}} = \Pi^{\mathrm{a}} \mathcal{J}^{\mathrm{a}} (\Pi^{\mathrm{a}})^t \, x, \qquad \xi^{\mathrm{r}} = \Pi^{\mathrm{r}} \mathcal{J}^{\mathrm{r}} (\Pi^{\mathrm{r}})^t \, x, \tag{10.2.16}$$

- On page 262, properties (10.5.6) should be

$$\begin{cases} \partial_x \mathcal{W}_I^{\mathrm{a}} \in L^2(0, \infty; W_2^{l-1}(\mathbb{R}^d)), \\ \partial_x \mathcal{W}_{II}^{\mathrm{a}} \in L^2(0, \infty; W_2^l(\mathbb{R}^d)). \end{cases} \tag{10.5.6}$$

Chapter 11

- On page 285, at line 9, x_0 should be $x_0 = \sup\{\, t \in [0, x_1],\ Y_k(t) = 0\,\}$.

- On page 294, at line 9, the comma before the inequality sign should be suppressed.

Chapter 12

- On page 306, in the last equation, $\Phi(\mathcal{Z}_{i+\tau})$ should be $\Phi(\mathcal{Z}^{i+\tau})$.

- On page 313, Reference [BS98] is from B. A. Bennett and M. D. Smooke.

Notes

These extra notes are intented to supplement the text and to answer various readers' questions.

Chapter 2

- For gases, it is preferable to use a kinetic theory description of the macroscopic equations since the associated thermodynamics in then valid out of equilibrium. However, it is possible to establish the consistency of kinetics thermodynamics and classical thermodynamics. This is done, in particular, in the book "The Mathematical Theory of Nonuniform Gases", S. Chapman and T. G. Cowling, Cambridge University Press, Section 2.431, page 41 of the 1995 edition.

- The velocity of the kth species can be written $v_k = v + V_k$ where the diffusion velocity of the kth species reads $V_k = F_k/\rho_k$. Therefore, the work per unit volume done by the force b_k acting on the kth species is $\rho_k v_k \cdot b_k = (\rho_k v + F_k) \cdot b_k$, and summing over the species we get the total work done per unit volume $\sum_k \rho_k v_k \cdot b_k = \sum_k (\rho_k v + F_k) \cdot b_k$. The term $\sum_k F_k \cdot b_k$ corresponds to the difference between the complete work term $\sum_k (\rho_k v + F_k) \cdot b_k$ and the average term $\sum_k \rho_k v \cdot b_k = \rho v b$ (with $\sum_k \rho_k b_k = \rho b$).

- On page 32, Notes 2.6, a very good history of the five births of fluid equations has been written by Oliver Darrigol in 'Between Hydrodynamics and Elasticity Theory: The First Five Births of the Navier-Stokes Equation', Arch. Hist. Exact Sci. 56 (2002) 95–150. The five first derivations of fluid equations are thus chronologically due to Navier(1822), Cauchy (1823), Poisson (1829), Saint-Venant (1837) and Stokes (1845) as detailes in Olivier Darrogil's paper.

Chapter 6

- Natural equilibrium points are independent of any reaction mechanism. Natural equilibrium point indeed only depend on the natural reaction vector space $\mathcal{R}$—that is, the largest reaction vector space—which is simply the orthogonal complement $\mathcal{R} = \mathcal{A}^\perp$ of the space spanned by the atom vectors $\mathcal{A}$.

- Continuous boltzmann gases have an entropy which explodes at 0K and cannot obey the third law. This was already pointed out for instance by L. C Woods, in the book 'The Thermodynamics of Fluid systems', Oxford Engineering Science Series 2, Clarendon Press Oxford, 1975, page 123. Note, however, that a boltzman gas is not a natural physical system, but simply a model gas towards which the Fermi and Bose gases converge at high temperatures. This problem neither arises with finite number of particles, quantum models, or cold crystals. In the JANAF themodynamic tables, it is also pointed out that cassical formulas should only be used above 10K and quantum models below 10K. Finally, absolute entropies for gases can conveniently be defined relative to absolute entropies for cold crystals ("The Properties of Gases and Liquids", R. C. Reid, J. M. Prausnitz, and B. E. Poling, McGraw-Hill Book Company, New-York, Fourth edition, 1986, page 152).

Chapter 7

- The temperature dependence of transport coefficients, discussed on pages 160—161 and Erratum, that is, the property that φ essentially behaves as $T^\varkappa$ with $1/2 \leq \varkappa \leq 1$, has important theoretical consequences.

Chapter 8

- In the proof of Theorem 8.5.5, after equation (8.5.12), we note that $\overline{\mathcal{T}}^*_{\scriptscriptstyle II} = \overline{\mathcal{T}}^*_{\scriptscriptstyle II}(W^*, \partial_x W^*_{\scriptscriptstyle II})$ since $V^{*\prime}_{\scriptscriptstyle II} = \phi^{-1}_{\scriptscriptstyle II}(W^*_{\scriptscriptstyle II})$.

Chapter 9

- In Proposition 9.6.1, Proposition 9.7.1, and Theorem 9.7.4, it is sufficient to consider the zero velocity case $\boldsymbol{v}^{\mathrm{e}} = 0$ thanks to galilean invariance. More specifically, if $\rho_1(t, \boldsymbol{x}), \ldots, \rho_n(t, \boldsymbol{x})$, $\boldsymbol{v}(t, \boldsymbol{x})$, $T(t, \boldsymbol{x})$, is a solution with $\boldsymbol{v}^{\mathrm{e}} = 0$, then $\rho_1(t, \boldsymbol{x} - \boldsymbol{v}^\infty t), \ldots, \rho_n(t, \boldsymbol{x} - \boldsymbol{v}^\infty t)$, $\boldsymbol{v}(t, \boldsymbol{x} - \boldsymbol{v}^\infty t) + \boldsymbol{v}^\infty$, $T(t, \boldsymbol{x} - \boldsymbol{v}^\infty t)$, is a solution with $\boldsymbol{v}^{\mathrm{e}} = \boldsymbol{v}^\infty$

Chapter 10

- Since the atomic species are part of the species, we have $\mathfrak{a}_{kl} = \delta_{kl}$, $k, l \in \mathfrak{A}$, and similarly, $\widetilde{\mathfrak{a}}_{kl} = \delta_{kl}$, $k, l \in \mathfrak{A}$.